Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

50X1-HUM

Title: THE GALAXY'S GRAVITATIONAL POTENTIAL by P. P. Parenago (Russia)

Source: Astronomicheskiy Zhurnal, Vol XXVII, No 6, Nov/Dec 1950, pp 329-340, Russian periodical (every 2 months)

CONFIDENTIAL

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

CONFIDENTIAL

50X1-HUM

THE GALAXY'S GRAVITATIONAL POTENTIAL

P. P. Parenago

State Astronomical

Institute, imeni Shternberg.

Submitted Feb '50.

[Abstract]

1. Introduction

Subject potential phi Φ is a function of the cylindrical coordinates R, z and relative to unit mass. If Φ were known, many problems of stellar astronomy could be solved. It will become known only when the density distribution of various galactic subsystems become known and Φ can be calculated as a density distribution summed over all subsystems. This method is preferred to any schematic representation of the Galaxy in the form of a compressed ellipsoid with a central nucleus (as many authors assume), or a disk with density falling off on both sides from the galactic plane. Undoubtedly, these calculations will be carried out in the future and Φ (R, z) will expressed in an analytical, graphical or tabular form.

Even now, however, one can make some conclusions and computations proceeding from other considerations. As yet, unfortunately, one has to limit oneself to finding the approximate expression of Φ for merely the galactic plane Φ (R, 0), and to allowing in an entirely approximate way for the influence of the z-coordinate, namely the distance from the galactic plane.

2. The Constants C_1, C_2, C_3 for a Planar Subsystem of Long-Periodic Cepheids.

The velocity of the center of gravity of various subsystem in their galactic rotation is

$$V_{\theta_0} = c_3 R / (c_1 + c_2 R^2)$$

CONFIDENTIAL

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

where the c's differ for different subsystems, and R is the distance from the Galaxy's axis of symmetry. The author's observations show that this formula holds well for long-periodic cepheids, that is for very flat subsystems and for globular stellar clusters (i.e. for typical spherical subsystems).

By employing the values $R_o = (7.2 \pm 0.2)$ kiloparsecs (i.e. the Sun's distance from the galactic center) and $V = (233 \pm 9)$ km/sec (i.e. the Sun's velocity), the author obtained improved values for the c's:

$$c_1 = 0.0032 \text{ sec}^2/\text{km}^2$$

$$c_2 = 0.000066 \text{ sec}^2/\text{km}^2 \cdot \text{kps}^2$$

$$c_3 = 0.246 \text{ sec/km} \cdot \text{kps}$$

3. The Constants K_1 and K_2 for the Circular Velocity

The constants in $V_{\theta_c} = k_1 R/(1 + k_2 R^2)$ are found to be:

$$k_1 = (72 \pm 7) \text{ km/sec} \cdot \text{kps}$$

$$k_2 = (0.0237 \pm 0.0030) \text{ kps}^{-2}.$$

4. Analytical Expression for the Galactic Potential in the Galactic Plane

The circular velocity is connected with potential thus:

$$V_{\theta_c}^2 = -R\frac{\partial \Phi}{\partial R}$$

Hence we find $\Phi = \dfrac{(K_1^2/2K_2)}{1 + K_2 R^2}$ by substituting $V_{\theta_c} = \dfrac{K_1 R}{1 + K_2 R^2}$ and integrating, assuming of course $\Phi = 0$ at infinity.

Obviously $\dfrac{K_1^2}{2K_2}$ is the value of the potential at the Galaxy's center, which is easily found now to be:

$$\Phi_c = \frac{K_1^2}{2K_2} = (11.0 \pm 2.6) \cdot 10^{14} \text{ cm}^2/\text{sec}^2$$

or better $= (11.0 \pm 1.5) \cdot 10^{14}$ cm^2/sec^2 (if we use certain refinements in the calculations).

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Therefore $\Phi = \frac{\Phi_c}{1+\kappa_2 R^2}$ cm^2/sec^2.

thus in the neighborhood of our Sun, we have $\Phi_0 = 4.93 \cdot 10^{14}$ cm^2/sec^2.

Now we can determine the parabolic velocity or velocity of escape for various parts of the galactic plane:

$$V_\infty = \sqrt{2\Phi}$$

Thus at the Galaxy's center, it equals 469 km/sec; near the Sun, it is 314 km/sec; and at the limits of the Galaxy (i.e. R = 13 kps) it is 210 km/sec. [Note: A table of numerical values of Φ, $d\Phi/dR$, $d^2\Phi/dR^2$, V_{θ_c}, V_∞, $V_\infty - V_{\theta_c}$, V_∞/V_{θ_c} is given versus R.]

5. The First Terms in the Series Expansion of Potential in the Galactic Plane

Obviously, we have the expansion:

$$\Phi = \Phi_0 + (R-R_0)\left(\frac{\partial\Phi}{\partial R}\right)_0 + \frac{1}{2}(R-R_0)^2\left(\frac{\partial^2\Phi}{\partial R^2}\right)_0 + \cdots$$

Hence we obtain:

$\Phi = 4.93 - 0.755\,(R-R_0) + 0.063\,(R-R_0)^2$, in units of 10^{14} cm^2/sec^2, by using $V_{\theta_c}^2 = -R\frac{\partial\Phi}{\partial R}$, its further derivatives, and our knowledge of the partial derivative at $R = R_0$.

6. Stability of Circular Orbits.

From numerical values the critical expression for stability turns out to be negative, in the neighborhood of the Sun, thus:

$$\left(\frac{\partial^2\Phi}{\partial R^2}\right)_0 + \frac{3}{R_0}\left(\frac{\partial\Phi}{\partial R}\right)_0 = -(1.98 \pm .36)\cdot 10^{-30}\ sec^{-2}.$$

In fact, it is always negative:

$$\frac{\partial^2\Phi}{\partial R^2} + \frac{3}{R}\frac{\partial\Phi}{\partial R} = -\frac{4\kappa_1^2}{(1+\kappa_2 R^2)^3}.$$

Hence the orbits are stable.

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

7. The Galactic Orbits of Stars

This is to be made a separate study.

8. Velocity of Escape in the Galaxy

[Note: The table mentioned in 4 is given here.]

9. Calculation of the Influence of Z upon Potential.

Obviously,

$$I(R,z) = I(R_0,0) + (R-R_0)\left(\frac{\partial\Phi}{\partial R}\right)_0 + z\left(\frac{\partial\Phi}{\partial z}\right)_0 + \cdots\cdots.$$

We get $I(R,z) = I(R,0) + 1/2 \left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 \cdot Z^2$, because $\left(\frac{\partial\Phi}{\partial z}\right)_0 = 0$ from the Galaxy's symmetry relative to the galactic plane and because $R \approx R_0$ and $(R-R_0) \approx 0$.

It is found [In Section 9] that $\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 = -2.50 \cdot 10^{-30}\ sec^{-2}$ in the neighborhood of the sun. Thus we have, in units of $10^{14}\ cm^2/sec^2$, near the sun:

$$I(R,z) = I(R,0) - 0.119z^2.$$

[Note: For 10 stars, the values of R, z, V, V_{∞} (velocity of escape) are given; for these stars z does not exceed 2 kps.]

10. Numerical Value of $\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{z=0}$

[Note: In this section, the numerical value of subject second derivative, employed in 9, is found from Poisson's equation of potential in cylindrical coordinates.]

11. Certain Considerations Concerning the Origin of Various Subsystems.

According to Ambartsumyan, planar subsystems originated at the outskirts of the Galaxy not far from the galactic plane, as shown by his investigations on stellar associations. Also, all elements making up planar and intermediate subsystems originated in stellar associations. The problem of the origin of the elements composing spherical subsystems has still not been solved.

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4

From $\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{z=0}$ we can find the period of oscillation P perpendicular to the galactic plane, since:

$$P = 2\pi/w, \; w^2 = -\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{z=0}$$, and $z = z_0 \sin \omega t$; here omega is angular velocity.

For the maximum value of z, namely z_0, it is expedient to take the mean value of z given by the quantity β from the following density formula:

$$D(z) = D_0 \cdot e^{-z/\beta}$$

Hence $\dot{z}_0 = \beta w$, namely the average velocity. Hence we can make the following table for the average velocity:

R	W sec^{-1}	P yrs	Planar subsystem (β = 50 ps)	Intermediate subsystem (β = 400 ps)	Spherical Subsystem (β = 200 ps)
0	$25.8 \cdot 10^{-15}$	7.10^6	46 km/sec	368 km/sec	1838 km/sec
R_0	1.58	126	2.4 km/sec	19 km/sec	97 km/sec
	0		0 " "	0 " "	0 " "

12. The Lowest Possible Limit of the Density of the Galactic Center.

From the formula

$$\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{\substack{z=0\\R=0}} = 2K_1^2 - 4\pi G \rho_c = -825 \cdot 10^{-30} \; sec^{-2}$$

(derived in Section 9, from Poisson's equation $\frac{\partial^2 \Phi}{\partial R^2} + \frac{\partial \Phi}{R \partial R} + \frac{\partial^2 \Phi}{\partial z^2} = -4\pi G \rho$ a) and from the fact that this second derivative is necessarily negative, we find:

$$\rho_{lim} = \frac{K_1^2}{2\pi G} = 1.30 \cdot 10^{23} \; gram/cm^3 \pm 20\%.$$

From other considerations we had $1.07 \cdot 10^{-21}$ gram/cm^3, which is 80 times greater.

For $D_c = 100 \, D_0 = 3.4 \cdot 10^{-55} \; cm^3$, the mean mass of the objects at the Galaxy's center turns out to be $p/D_c = 1.5$ times the sun's mass.

- E N D -

Declassified in Part - Sanitized Copy Approved for Release 2012/06/08 : CIA-RDP82-00039R000100110014-4